International Journal of Theoretical Physics, Vol. 32, No. 5, 1993

More about Superconformal Field Theory. Hamiltonian Formalism

A. Foussats,[1] C. Repetto,[1] O. P. Zandron,[1] and O. S. Zandron[1]

Received March 30, 1992

The Hamiltonian formalism for the $N=1$, $d=4$ superconformal system is given. The first-order formalism is found by starting from the canonical covariant one. As the conformal supergravity is a higher-derivative theory, to analyze the second-order Hamiltonian formalism the Ostrogradski transformation is introduced to define canonical momenta.

1. INTRODUCTION

The properties of conformal $N=1$, $d=4$ supergravity were given in Kaku *et al.* (1978) and van Nieuwenhuizen (1981). This was done starting from the supersymmetric extension of the Weyl action $R^2_{\mu\nu}-\frac{1}{3}R^2$. In that work the constraints and the transformation rules for the gauge fields are analyzed. Moreover, the action quadratic in the curvatures is found. Among other results the following two features of the conformal supergravity are made clear: (i) the gauge algebra closes off shell; (ii) the conformal model of supergravity shows that local supersymmetry can exist in flat space-time.

Other important papers about conformal supergravity theories in diverse dimensions are collected in Salam and Sezgin (1989).

The conformal $N=1$, $d=4$ supergravity was also analyzed from the point of view of the group manifold approach (Castellani *et al.*, 1981, 1991). Castellani *et al.* (1981) showed that it is possible to write a rheonomic action in the whole group manifold G, linear in the curvatures, which reproduce the same results obtained in Kaku *et al.* (1978) and van Nieuwenhuizen (1981) when the theory is restricted to the space-time.

There is not an equivalent situation in $d=3$ because, as shown in van Nieuwenhuizen (1985), in this case the action for the conformal supergravity

[1]Facultad de Ciencias Exactas Ingeniería y Agrimensura de la U.N.R., 2000 Rosario, Argentina.

0020-7748/93/0500-0775$07.00/0

is given by supersymmetric extension of the Lorentz–Chern–Simons theory.

On the other hand, it is well known to be convenient to study these gauge theories from the canonical point of view. The canonical method allows one to separate unambiguously the physical degrees of freedom from the gauge ones, once all the constraints are known. Moreover, there are several reasons for considering the canonical formalism as more adequate with regard to the quantization of a supersymmetric gauge field theory. Many papers have devoted considerable attention to the construction of a Hamiltonian formalism for gravity and ordinary Poincaré supergravity (OPS) (Deser *et al.*, 1977; Nelson and Teitelboim, 1977, 1978; Pilati, 1978; Castellani *et al.*, 1982; Henneaux, 1983, 1986; Diaz, 1986).

In previous work the canonical covariant formalism (CCF) was formulated and later applied to several cases (D'Adda *et al.*, 1985; Lerda *et al.*, 1985, 1987; Foussats and Zandron, 1990, 1991). All these applications were given in the OPS. The OPS does not have the two properties (i) and (ii) mentioned above. Furthermore, the conformal supergravities are higher-derivative theories, which constitute an interesting subject from the theoretical point of view. The lack of attention devoted to the conformal supersymmetry may be due to the underdeveloped knowledge on how to treat higher-derivative field theory rather than to a lack of physical interest. Therefore, it is interesting to analyze in the canonical framework the constraints of a superconformal dynamical system and to find the total Hamiltonian as the generator of the time evolution of generic functionals. Consequently, the motivation of the present paper is essentially based on the properties of the conformal supergravities.

In a recent paper (Foussats *et al.*, 1992) the first-order CCF for the $d=3$ superconformal Chern–Simons theory was constructed. The second-order formalism was also developed. The second-order formalism, where the higher-derivative character of the theory becomes apparent, is described by a singular Lagrangian density containing higher-derivative terms (van Nieuwenhuizen, 1985). The construction of the second-order Hamiltonian formalism is nontrivial and it is very different from the case of the OPS (Deser *et al.*, 1977; Nelson and Teitelboim, 1977, 1978; Pilati, 1978; Castellani *et al.*, 1982; Henneaux, 1983, 1986; Diaz, 1986; Foussats and Zandron, 1990). It is necessary to perform an Ostrogradski transformation to introduce canonical momenta (Nesterenko, 1989; Li, 1990, 1991; Li and Xin, 1991). The purpose of the present paper is to construct the proper Hamiltonian formalism for the $N=1$, $d=4$ conformal supergravity starting from the first-order CCF. Moreover, we will show the method to treat the theory in the second-order Hamiltonian formalism. The paper is organized as follows: In Section 2, the definitions and the main results of the Lagrangian formulation on the group manifold are reviewed. In Section 3, starting from the canonical covariant formalism, the set of constraints and the total Hamiltonian of the system is found. In Section 4, the space-time decomposition is performed and the Hamiltonian as the generator of time evolutions is written as linear combination of first-class constraints. Finally, in Section 5, the method to construct the second-order formalism is analyzed.

2. DEFINITIONS AND PRELIMINARIES

To construct the CCF, we start from the Lagrangian formalism on the group manifold developed in Castellani *et al.* (1981). We briefly recall the results obtained in that paper that we are going to use later on. The conformal $N=1$ supergravity in $d=4$ dimensions is based on the superalgebra $\mathscr{G}$ associated to the supergroup manifold $G=SU(2, 2/1)$. The 24 generators $\mathbf{T}_A$ of local symmetries associated to this superconformal group are

$$\mathbf{T}_A=(P_a, M_{ab}, K_a, D, A, Q_\alpha, S_\alpha) \tag{2.1}$$

and the corresponding 1-form gauge fields are

$$\mu^A=(V^a, \omega^{ab}, K^a, D, A, \xi^\alpha, \varphi^\alpha) \tag{2.2}$$

The fundamental $\mathscr{G}$-valued 1-form is written

$$\mu=V^aP_a+\omega^{ab}M_{ab}+K^aK_a+DD+AA+\bar{\xi}^\alpha Q_\alpha+\bar{\varphi}^\alpha S_\alpha \tag{2.3}$$

and the 2-form curvatures remain defined by $R(\mu)=d\mu+\mu\wedge\mu$ or, in components,

$$R^A(\mu)=d\mu^A-\tfrac{1}{2}C_{BC}{}^A\mu^B\wedge\mu^C \tag{2.4}$$

The choice of the bosonic gauge subgroup $H\subset G$ is not unique. In Castellani *et al.* (1981) it is taken as $H=(M, D, A)$ and this parametrization correctly reproduces the space-time theory developed in Kaku *et al.* (1978) and van Nieuwenhuizen (1981). Another important result in the group manifold approach is that the correct Lagrangian density is linear in the curvatures $R^A(\mu)$. There is a one-parameter family of Lagrangian densities and the cohomology part of the total Lagrangian is equivalent, up to partial integration, to the Lagrangian quadratic in curvatures given in Kaku *et al.* (1978) and van Nieuwenhuizen (1981). In the space-time approach, the duality relation between the curvatures of the dilaton D and the axial gauge field A holds off-shell. In the group manifold approach the duality relation is obtained by adding to the cohomology Lagrangian $\mathscr{L}$(cohom) a Lagrangian $\mathscr{L}$(Maxwell). As is well known, in the group manifold this piece of Lagrangian is constructed by using the 2-form curvatures $R(A)$ and $R(D)$ and introducing two 0-forms F_{ab} and G_{ab}. After the equations of motion are solved, these two nongeometrical objects are identified respectively with the inner–inner components of the curvatures $R(A)$ and $R(D)$. In this way the duality relation in the group manifold formalism is obtained. Finally, to obtain a higher-derivative theory as is required by a conformal theory of supergravity, another piece of Lagrangian must be added. This piece, called $\mathscr{L}$(constraint), is constructed by means of two 1-form fields: t^{ab} (bosonic Lagrange multiplier) and λ (fermionic Lagrange multiplier). Thus, the final constraints in the first-order formalism on the group manifold are field equations of motion. Therefore, our starting point is the total Lagrangian density:

$$\mathscr{L} = \mathscr{L}(\text{cohom}) + \mathscr{L}(\text{Maxwell}) + \mathscr{L}(\text{constraint}) \tag{2.5}$$

whose explicit expression was given in equations (5.5)–(5.8) of Castellani *et al.* (1981), and thus we do not rewrite it here.

So, in the first-order formalism on the group manifold the independent dynamical fields are

$$\tilde{\mu}^A = (\mu^A, F^{ab}, G^{ab}, t^{ab}, \lambda) \tag{2.6}$$

i.e., the seven 1-form gauge fields μ^A; the two bosonic 0-forms F^{ab} and G^{ab} (Lorentz tensors); the 1-form t^{ab} (bosonic Lorentz tensor); and the 1-form λ (Majorana spinor). The physical fields of the theory are the graviton V^a, the gravitino ξ^a, and the axion A. The rheonomic Lagrangian (2.5) gives rise to the equations of motion for the conformal supergravity, which are solved for the rheonomic solution for the curvatures given in Castellani *et al.* (1981).

Finally, the conformal gauge field K^a, the conformal gravitino φ, and the Lagrange multipliers t^{ab} and λ are expanded in the coframe (V^a, ξ). Therefore, in the group manifold approach it remains clear that the conformal directions K^a and φ are not independent, but are expanded themselves in the coframe of the physical superspace G/H.

These are some of the results obtained in Castellani *et al.* (1981) which we will use in the construction of the first-order CCF.

3. THE FIRST-ORDER CCF. CONSTRAINTS AND TOTAL HAMILTONIAN

First, we recall that there are two different versions in which we can formulate the CCF (Foussats and Zandron, 1990):

(a) A version valid in the cases in which the supergravity is described by a linear Lagrangian density in the curvatures.
(b) A version valid in the cases in which the supergravity is described by a polynomial Lagrangian in the curvatures.

In the first-order formalism the Lagrangian density on the group manifold is linear in the curvatures and corresponds to version (a).

The corresponding 11 momenta $\tilde{\Pi}_A$ that are canonical conjugate to the dynamical fields (2.6) are obtained by functional variation of the Lagrangian density (2.5) with respect to the velocities $d\tilde{\mu}^A$. The canonical momenta do not depend on the velocities; therefore these relationships define primary constraints. They are as follows:

$$\Phi_{ab}(\omega)=\pi_{ab}(\omega)\approx 0 \tag{3.1a}$$

$$\Phi_a(V)=\pi_a(V)-(\omega^{bc}\wedge K^d+t^{bc}\wedge V^d)\varepsilon_{abcd}+\tfrac{1}{2}\bar{\lambda}\wedge\gamma_5\gamma_a\xi\approx 0 \tag{3.1b}$$

$$\Phi_a(K)=\pi_a(K)-\omega^{bc}\wedge V^d\varepsilon_{abcd}\approx 0 \tag{3.1c}$$

$$\Phi(A)=\pi(A)+(1-\tfrac{1}{3}y)(\tfrac{3}{4}i\bar{\xi}\wedge\varphi+\tfrac{3}{8}F^{ab}V^c\wedge V^d\varepsilon_{abcd})-iyV^a\wedge K_a\approx 0 \tag{3.1d}$$

$$\Phi(D)=\pi(D)-\tfrac{1}{2}(1+y)(\bar{\xi}\wedge\gamma_5\varphi-G^{ab}V^c\wedge V^d\varepsilon_{abcd})\approx 0 \tag{3.1e}$$

$$\Phi(\xi)=\pi(\xi)-\gamma_5\gamma_a\xi\wedge K^a-\gamma_5\gamma_a\lambda\wedge V^a\approx 0 \tag{3.1f}$$

$$\Phi(\varphi)=\pi(\varphi)+\gamma_5\gamma_a\varphi\wedge V^a\approx 0 \tag{3.1g}$$

$$\Phi_{ab}(F)=\tau_{ab}(F)\approx 0 \tag{3.1h}$$

$$\Phi_{ab}(G)=\tau_{ab}(G)\approx 0 \tag{3.1i}$$

$$\Phi(\lambda)=\theta(\lambda)\approx 0 \tag{3.1j}$$

$$\Phi_{ab}(t)=\theta_{ab}(t)\approx 0 \tag{3.1k}$$

The canonical Hamiltonian is defined by $H_{\mathrm{can}}=d\tilde{\mu}^A\wedge\tilde{\Pi}_A-\mathscr{L}$ and in the present case we find by direct computation

$$\begin{aligned}
H_{\mathrm{can}}=\;&\omega^{ae}\wedge\omega_e{}^b\wedge V^c\wedge K^d\varepsilon_{abcd}-2V^a\wedge V^b\wedge K^c\wedge K^d\varepsilon_{abcd}\\
&-\bar{\xi}\wedge\sigma^{ab}\varphi\wedge V^c\wedge K^d\varepsilon_{abcd}+(1+y)\bar{\xi}\wedge\gamma_5\varphi\wedge V^a\wedge K_a\\
&-\tfrac{1}{2}(1-y)\bar{\xi}\wedge\gamma_5\varphi\wedge\bar{\xi}\wedge\varphi+\tfrac{1}{4}\omega^{ab}\wedge\bar{\xi}\wedge\gamma^c\xi\wedge K^d\varepsilon_{abcd}\\
&-\tfrac{1}{4}\omega^{ab}\wedge\bar{\varphi}\wedge\gamma^c\varphi\wedge V^d\varepsilon_{abcd}+\tfrac{3}{4}iA\wedge\bar{\xi}\wedge\gamma_b\xi\wedge K^b\\
&+\tfrac{3}{4}iA\wedge\bar{\varphi}\wedge\gamma_b\varphi\wedge V^b-\bar{\varphi}\wedge\gamma_5\gamma_a\gamma_b\xi\wedge V^a\wedge K^b\\
&-\bar{\xi}\wedge\gamma_5\gamma_a\gamma_b\varphi\wedge K^a\wedge V^b+2\bar{\xi}\wedge\gamma_5\sigma_{ab}\varphi\wedge V^a\wedge K^b\\
&-\tfrac{1}{8}(1-y)\bar{\xi}\wedge\sigma_{ab}\xi\wedge\bar{\varphi}\wedge\sigma_{cd}\varphi\varepsilon^{abcd}\\
&-[\tfrac{1}{32}(1-\tfrac{1}{3}y)F_{ab}F^{ab}+\tfrac{1}{24}(1+y)G_{ab}G^{ab}]V^c\wedge V^d\wedge V^e\wedge V^f\varepsilon_{cdef}\\
&+\tfrac{3}{8}i(1-\tfrac{1}{3}y)F^{ab}\bar{\xi}\wedge\gamma_5\varphi\wedge V^c\wedge V^d\varepsilon_{abcd}\\
&-\tfrac{1}{2}(1+y)G^{ab}(\tfrac{1}{2}\bar{\xi}\wedge\varphi+2V^e\wedge K_e)\wedge V^c\wedge V^d\varepsilon_{abcd}
\end{aligned}$$

$$+\omega^{ae}\wedge V_e\wedge t^{bc}\wedge V^d\varepsilon_{abcd}-D\wedge V^a\wedge t^{bc}\wedge V^d\varepsilon_{abcd}$$
$$+\tfrac{1}{4}\bar{\xi}\wedge\gamma^a\xi\wedge t^{bc}\wedge V^d\varepsilon_{abcd}+\tfrac{1}{4}\omega^{ae}\wedge\bar{\lambda}\wedge\gamma^c\xi\wedge V^d\varepsilon_{abcd}$$
$$-D\wedge\bar{\xi}\wedge\gamma_5\gamma_a\lambda\wedge V^a-\tfrac{1}{8}\bar{\lambda}\wedge\gamma_5\gamma_a\xi\wedge\bar{\xi}\wedge\gamma^a\xi$$
$$+\tfrac{3}{4}iA\wedge\bar{\xi}\wedge\gamma_a\lambda\wedge V^a+\bar{\varphi}\wedge\gamma_5\gamma_a\gamma_b\lambda\wedge V^a\wedge V^b \tag{3.2}$$

The total Hamiltonian H_T (bosonic 4-form), which is a first-class dynamical quantity in the Dirac sense, is defined by

$$H_T=H_{\text{can}}+\Lambda^A(\tilde{\mu})\wedge\Phi_A(\tilde{\mu}) \tag{3.3}$$

where Λ^A are arbitrary Lagrange multipliers which can be determined, and given by $\Lambda^A(\tilde{\mu})=d\tilde{\mu}^A$.

When the properties (2.1) and (2.2) of Foussats and Zandron (1990) for the form-brackets are used, it is possible to show that the form-brackets between constraints give rise to expressions different from zero. That is to say, the primary constraints Φ_A are of second class. The condition of preservation of the primary constraints (or Dirac's consistency condition) in the CCF is given by

$$d\Phi_A=(\Phi_A, H_T)\approx 0 \tag{3.4}$$

When the form-brackets appearing in (3.4) are explicitly computed, we arrive at

$$d\Phi_A=-[\text{field equations of motion}]+(\Phi_A, \Lambda^B)\wedge\Phi_B\approx 0 \tag{3.5}$$

These are very important equations because they imply that there are no secondary constraints in the CCF.

The formalism is completed by introducing the fundamental dynamical equation $dA=(A, H_T)+\partial A$, where $A=A(\tilde{\mu}, \tilde{\Pi})$ is a generic polynomial in the canonical variables (Foussats and Zandron, 1991). Consequently, according to (3.4), when $A=\Phi_A$ the field equations of motion in the framework of the CCF are obtained. We notice that when we put $A=R^A(\mu)$, the above dynamical equation gives rise to the Bianchi identities:

$$dR^A-(R^A, H_T)-\partial R^A=0 \tag{3.6}$$

The field equations of motion for the different values of the compound index A are

$$d\Phi_{ab}(t)=-[R^c(V)\wedge V^d\varepsilon_{abcd}]\approx 0 \tag{3.7a}$$
$$d\Phi(\lambda)=-[\gamma_5\gamma_a R(Q)\wedge V^a-\tfrac{1}{2}\gamma_5\gamma_a\xi\wedge R^a(V)]\approx 0 \tag{3.7b}$$
$$d\Phi_{ab}(F)=-[\tfrac{1}{16}F_{ab}V^c\wedge V^d\wedge V^e\wedge V^f\varepsilon_{cdef}-\tfrac{3}{8}R(A)V^c\wedge V^d\varepsilon_{abcd}]\approx 0 \tag{3.7c}$$

$$d\Phi_{ab}(G) = -[\tfrac{1}{12}G_{ab}V^c \wedge V^d \wedge V^e \wedge V^f \varepsilon_{cdef} - \tfrac{1}{2}R(D)V^c \wedge V^d \varepsilon_{abcd}] \approx 0 \tag{3.7d}$$

$$\begin{aligned} d\Phi(D) = &-\tfrac{1}{2}(1+y)[-D^H G^{ab} \wedge V^c \wedge V^d \varepsilon_{abcd} - \tfrac{1}{2}G^{ab}\bar{\xi} \wedge \gamma^c \xi \wedge V^d \varepsilon_{abcd} \\ &-R^a(V)G^{bc} \wedge V^d \varepsilon_{abcd} + \bar{R}(Q) \wedge \gamma_5 \varphi - \bar{R}(S) \wedge \gamma_5 \xi] \\ &+t^{ab} \wedge V^c \wedge V^d \varepsilon_{abcd} + \bar{\lambda} \wedge \gamma_5 \gamma_a \xi \wedge V^a \approx 0 \end{aligned} \tag{3.7e}$$

$$\begin{aligned} d\Phi(A) = &-\{-\tfrac{3}{4}(1-\tfrac{1}{3}y)(D^H F^{ab} \wedge V^c \wedge V^d \varepsilon_{abcd} - \tfrac{1}{4}F^{ab}\bar{\xi} \wedge \gamma^c \xi \wedge V^d \varepsilon_{abcd}) \\ &+iy[R^a(V) \wedge K_a - R^a(K) \wedge V_a] \\ &-\tfrac{3}{4}i(1-\tfrac{1}{3}y)[\bar{R}(Q) \wedge \varphi - \bar{R}(S) \wedge \xi] \\ &-\tfrac{3}{4}i\bar{\lambda} \wedge \gamma_b \xi \wedge V^b - \tfrac{3}{4}(1-\tfrac{1}{3}y)R^a(V)F^{bc} \wedge V^d \varepsilon_{abcd}\} \approx 0 \end{aligned} \tag{3.7f}$$

$$\begin{aligned} d\Phi_{ab}(\omega) = &-\{[R^c(V) \wedge K^d + R^c(K) \wedge V^d]\varepsilon_{abcd} \\ &-\tfrac{1}{4}\bar{\lambda} \wedge \gamma^c \xi \wedge V^d \varepsilon_{abcd} + t^{cd} \wedge V^e \wedge V_{[a}\varepsilon_{b]cde}\} \approx 0 \end{aligned} \tag{3.7g}$$

$$\begin{aligned} d\Phi_a(V) = &-[R^{bc} \wedge K^d \varepsilon_{bcda} - iyR(A) \wedge K_a - \bar{R}(S)\gamma_5\gamma_a \wedge \varphi \\ &+(1+y)G^{bc}K_a \wedge V^d \wedge V^e \varepsilon_{bcde} + \tfrac{1}{2}\bar{\lambda} \wedge \gamma_5\gamma_a R(Q) \\ &-\bar{\lambda} \wedge \gamma_5\gamma_b\gamma_a\varphi \wedge V^b + \tfrac{1}{2}\bar{\lambda} \wedge \gamma_5\gamma_a\gamma_b\varphi \wedge V^b \\ &-\tfrac{1}{4}t^{bc} \wedge \bar{\xi} \wedge \gamma^d \xi \varepsilon_{bcda} + \tfrac{1}{2}D^H\lambda \wedge \gamma_5\gamma_a\xi \\ &+D^H t^{bc} \wedge V^d \varepsilon_{bcda}] \approx 0 \end{aligned} \tag{3.7h}$$

$$\begin{aligned} d\Phi_a(K) = &-[R^{bc}(M) \wedge V^d \varepsilon_{abcd} - \bar{R}(Q) \wedge \gamma_5\gamma_a\xi - iyR(A) \wedge V_a \\ &-(1+y)G^{bc}V_a \wedge V^d \wedge V^e \varepsilon_{bcde}] \approx 0 \end{aligned} \tag{3.7i}$$

$$\begin{aligned} d\Phi(\xi) = &-\{\tfrac{1}{2}(1+y)\gamma_5\varphi \wedge R(D) - \tfrac{3}{4}i(1-\tfrac{1}{3}y)\varphi \wedge R(A) \\ &-[\tfrac{3}{8}i(1-\tfrac{1}{3}y)\gamma_5\varphi F^{ab} - \tfrac{1}{4}(1+y)\varphi G^{ab}] \wedge V^c \wedge V^d \varepsilon_{abcd} \\ &-\tfrac{1}{2}\gamma^a\xi \wedge t^{bc} \wedge V^d \varepsilon_{abcd} + \tfrac{1}{4}\gamma^a\xi \wedge \bar{\lambda} \wedge \gamma_5\gamma_a\xi \\ &-\tfrac{1}{2}\gamma_5\gamma_a\lambda \wedge R^a(V) - \tfrac{1}{4}\gamma_5\gamma_a\lambda \wedge \bar{\xi} \wedge \gamma^a\xi + 2\gamma_5\gamma_a R(Q) \wedge K^a \\ &-\gamma_5\gamma_a\xi R^a(K) + \gamma_5\gamma_a D^H\lambda \wedge V^a\} \approx 0 \end{aligned} \tag{3.7j}$$

$$\begin{aligned} d\Phi(\varphi) = &-\{-\tfrac{1}{2}(1+y)\gamma_5\xi \wedge R(D) + \tfrac{3}{4}i(1-\tfrac{1}{3}y)\xi \wedge R(A) \\ &+[\tfrac{3}{8}i(1-\tfrac{1}{3}y)\gamma_5\xi F^{ab} - \tfrac{1}{4}(1+y)\xi G^{ab}] \wedge V^c \wedge V^d \varepsilon_{abcd} \\ &-2\gamma_5\gamma_a R(S) \wedge V^a + \gamma_5\gamma_a\varphi \wedge R^a(V) \\ &-2\gamma_5\sigma_{ab}\lambda \wedge V^a \wedge V^b\} \approx 0 \end{aligned} \tag{3.7k}$$

where D^H is the exterior covariant derivative in $H \subset G$.

To find equations (3.7), the properties (2.1) and (2.2) given in Foussats and Zandron (1990) for the form-brackets were repeatedly used.

Looking at equations (3.7a) and (3.7b), we see that they are the constraints on the curvatures $R^a(V)$ and $R(Q)$ demanded in the conformal supergravities. The Maxwell equations (3.7c) and (3.7d) give respectively the rheonomic solutions for the curvatures $R(A)$ and $R(D)$.

We complete the section by showing an alternative canonical method. The CCF also can be given by starting from an equivalent Lagrangian density. As shown in Castellani *et al.* (1981), by using the duality relation $R_{\mu\nu}(D) = -\frac{1}{4}i\,{}^*R_{\mu\nu}(A)$ valid in the off-shell space-time, we can write the group manifold Lagrangian density (2.5)

$$\mathscr{L} = R^{ab}(M) \wedge R^{cd}(M)\varepsilon_{abcd} - 8\bar{R}(Q) \wedge \gamma_5 R(S) + 4iR(A) \wedge R(D) \quad (3.8)$$

This Lagrangian density, quadratic in the curvatures, is unique because the y dependence disappears. To construct the CCF from (3.8), we must use version (b) of the formalism (Foussats and Zandron, 1990). The treatment is rather different. The first step is to write the Lagrangian as a second-order polynomial in the variables $\Lambda^A = d\mu^A$ instead of using the curvatures, and treat the dynamical fields μ^A and Λ^A as independent variables. Therefore, we must add to the resulting Lagrangian a set of constraints $(\Lambda^A - d\mu^A)$, with the corresponding arbitrary Lagrange multipliers β_A to be determined. Consequently, the Lagrangian density can be written as follows:

$$\mathscr{L} = v + \Lambda^A \wedge v_A + \tfrac{1}{2}\Lambda^A \wedge \Lambda^B \wedge v_{AB} + (\Lambda^A - d\mu^A) \wedge \beta_A \quad (3.9)$$

Now, the independent field variables are μ^A, Λ^A, and β_A. For each one of these variables the corresponding canonical conjugate momenta must be defined. According to the results obtained in Foussats and Zandron (1990) it is also possible to prove in the present case that:

(i) All the relationships between fields and momenta determine primary constraints and none of them are first class.

(ii) The total Hamiltonian has a similar expression to that given in (3.3), but in this case $\Phi_A = \Pi_A + \beta_A$ and Π_A are the canonical momenta of the field variables $\mu^A = (\omega^{ab}, V^a, K^a, D, A, \xi, \varphi)$ which appear in the Lagrangian (3.9). The other primary constraints do not appear in the final expression of the total Hamiltonian. Moreover, $dH_T = (H_T, H_T) = 0$, i.e., the first-class dynamical quantity H_T is strongly conserved.

In the higher-curvature supergravity the canonical Hamiltonian is given by $H_{\text{can}} = -v + \frac{1}{2}\Lambda^A \wedge \Lambda^B \wedge v_{AB}$ (Foussats and Zandron, 1990), which in this case is written

$$\begin{aligned} H_{\text{can}} = {} & \tfrac{1}{2}\Lambda^{ab}(M) \wedge \Lambda^{cd}(M)\varepsilon_{abcd} - 4\bar{\Lambda}(Q) \wedge \gamma_5\Lambda(S) + 2i\Lambda(A) \wedge \Lambda(D) \\ & - \tfrac{1}{2}C^{ab}(M) \wedge C^{cd}(M)\varepsilon_{abcd} + 4\bar{C}(Q) \wedge \gamma_5 C(S) - 2iC(A) \wedge C(D) \quad (3.10) \end{aligned}$$

where

$$C^{ab}(M) = -\omega^{ac} \wedge \omega_c{}^b + 2(V^a \wedge K^b - V^b \wedge K^a) + \bar{\xi} \wedge \sigma^{ab}\varphi \tag{3.11a}$$

$$C(A) = i\bar{\xi} \wedge \gamma_5\varphi \tag{3.11b}$$

$$C(D) = -2V^a \wedge K_a - \tfrac{1}{2}\bar{\xi} \wedge \varphi \tag{3.11c}$$

$$\bar{C}(Q) = \tfrac{1}{2}\omega^{ab} \wedge \bar{\xi}\sigma_{ab} - \tfrac{1}{2}\bar{\xi} \wedge D + \tfrac{3}{4}i\bar{\xi}\gamma_5 \wedge A - \bar{\varphi}\gamma_a \wedge V^a \tag{3.11d}$$

$$C(S) = -\tfrac{1}{2}\omega^{ab} \wedge \sigma_{ab}\varphi - \tfrac{1}{2}D \wedge \varphi + \tfrac{3}{4}iA \wedge \gamma_5\varphi + K^a \wedge \gamma_a\xi \tag{3.11e}$$

We do not complete the explicit computation for this version because it is straightforward and of course the results are equivalent to those already obtained.

4. SPACE-TIME DECOMPOSITION

Now, we must find the proper Hamiltonian $\tilde{\mathscr{H}}$ as the generator of the time evolution of generic functionals. This is carried out by choosing a privileged time direction in the coset manifold $M = G/H$ (physical superspace), losing the manifest covariance of the formalism (Foussats and Zandron, 1991). Therefore, to relate the first-order CCF with the canonical component formalism (CVF), we must first consider all the forms written in the holonomic basis. Thus, in particular, the gauge fields are written $\mu^A = \mu^A_\nu\, dx^\nu$ ($\nu = 0, 1, 2, 3$). Moreover, the form-brackets introduced in the CCF must be related to the Poisson brackets defined in the usual CVF according to equation (3.6) of Foussats and Zandron (1991). Second, we must consider fields and forms defined on a spacelike $x^0 = t = t^0$ hypersurface Σ of three dimensions. In the CCF, the time component of the momenta does not appear when they are restricted to the three-dimensional hypersurface Σ after the time variable is chosen. So, the momenta $\Pi_A(\mu)$ and the components $\Pi_A{}^i$ ($i = 1, 2, 3$) are related by the equation

$$\Pi_A = g^{-1/2}\Pi_A{}^i(x)\,\varepsilon_{ijk}\, dx^j \wedge dx^k \tag{4.1}$$

A similar expression holds for the corresponding constraints Φ_A.

In the space-time decomposition (Arnowitt *et al.*, 1982) it is useful to introduce the functions N_i and $N^\perp$, which are respectively the shift and lapse functions and they determine the components of the four-dimensional metric tensor $g_{\mu\nu}$. The vierbein's holonomic components ${}^4e_{a\mu}$ split according to

$$\begin{gathered} {}^4e_{ai} = {}^3e_{ai} = e_{ai}, \qquad {}^3e_a{}^i = e_a{}^i, \qquad {}^4e_a{}^i = {}^3e_a{}^i + (N^\perp)^{-1}N^i n_a \\ e_a{}^i e_{bi} = \eta_{ab} + n_a n_b \end{gathered} \tag{4.2}$$

where $n_a = n^\mu\, {}^4e_{a\mu}$ is the normal to the hypersurface Σ.

Once this is done, the Hamiltonian $\tilde{\mathscr{H}}$ (bosonic 3-form) remains defined from the Hamiltonian (3.3) by means of the equation

$$\int H_T = \int dx^0 \wedge \tilde{\mathscr{H}} \tag{4.3}$$

where

$$\tilde{\mathscr{H}} = \int [\tfrac{1}{2}\omega_0{}^{ab}\mathscr{H}_{ab}(x) + L^a{}_0\mathscr{H}^L_a(x) + K^a{}_0\mathscr{H}^K_a(x) + \bar{\xi}_0\mathscr{H}^\xi(x) + \bar{\varphi}_0\mathscr{H}^\varphi(x)$$
$$+ D_0\mathscr{H}^D(x) + A_0\mathscr{H}^A(x) + t_0^{ab}h_{ab}(x) + \bar{\lambda}_0 h(x)]\, d^3x \tag{4.4}$$

and

$$\begin{aligned}\mathscr{H}_{ab}(x)\, d^3x = [&\text{field equation of motion}\\ &+ [\psi_a(V)\wedge V_b - \psi_b(V)\wedge V_a]\\ &+ \{\psi_a(K)\wedge K_b - \psi_b(K)\wedge K_a\} + 2(\omega_a{}^c\wedge\psi_{cb} - \omega_b{}^c\wedge\psi_{ca})\\ &- \bar{\xi}\wedge\sigma_{ab}\psi(\xi) - \bar{\varphi}\wedge\sigma_{ab}\psi(\varphi)]_{|\Sigma}\approx 0\end{aligned} \tag{4.5a}$$

$$\begin{aligned}\mathscr{H}^L_a(x)\, d^3x = [&\text{field equation of motion}\\ &- \omega^b{}_a\wedge\psi_b(V) + D\wedge\psi_a(V)\\ &- 4K^b\wedge\psi_{ab} + 2K_a\wedge\psi(D) - \bar{\varphi}\wedge\gamma_a\psi(\xi)]_{|\Sigma}\approx 0\end{aligned} \tag{4.5b}$$

$$\begin{aligned}\mathscr{H}^K_a(x)\, d^3x = [&\text{field equation of motion}\\ &+ \omega^b{}_a\wedge\psi_b(K) - D\wedge\psi_a(K)\\ &- 4V^b\wedge\psi_{ab} - 2V_a\wedge\psi(D) + \bar{\xi}\wedge\gamma_a\psi(\varphi)]_{|\Sigma}\approx 0\end{aligned} \tag{4.5c}$$

$$\begin{aligned}\mathscr{H}^\xi(x)\, d^3x = [&\text{field equation of motion}\\ &+ \tfrac{1}{2}\gamma^a\xi\wedge\psi_a(V) + \tfrac{1}{2}\varphi\wedge\psi(D)\\ &+ (\tfrac{1}{2}D + \tfrac{1}{2}\omega^{ab}\sigma_{ab} - \tfrac{3}{4}i\gamma_5 A)\wedge\psi(\xi) - \sigma^{ab}\varphi\wedge\psi_{ab}\\ &- i\gamma_5\varphi\wedge\psi(A) - \gamma_a K^a\wedge\psi(\varphi)]_{|\Sigma}\approx 0\end{aligned} \tag{4.5d}$$

$$\begin{aligned}\mathscr{H}^\varphi(x)\, d^3x = [&\text{field equation of motion}\\ &- \tfrac{1}{2}\gamma^a\varphi\wedge\Phi_a(K) + \tfrac{1}{2}\xi\wedge\psi(D)\\ &- (\tfrac{1}{2}D - \tfrac{1}{2}\omega^{ab}\sigma_{ab} - \tfrac{3}{4}i\gamma_5 A)\wedge\psi(\varphi) - \sigma^{ab}\xi\wedge\psi_{ab}\\ &+ i\gamma_5\xi\wedge\psi(A) + \gamma_a V^a\wedge\psi(\xi)]_{|\Sigma}\approx 0\end{aligned} \tag{4.5e}$$

$$\mathscr{H}^D(x)\, d^3x = -\mathbf{D}\boldsymbol{\pi}(D)\approx 0 \tag{4.5f}$$

$$\mathscr{H}^A(x)\, d^3x = -\mathbf{D}\boldsymbol{\pi}(A)\approx 0 \tag{4.5g}$$

$$h_{ab}(x)\, d^3x = [\text{field equation of motion}]_{|\Sigma}\approx 0 \qquad 4.5h)$$

$$h(x)\, d^3x = [\text{field equation of motion}]_{|\Sigma}\approx 0 \tag{4.5i}$$

In equations (4.5f) and (4.5g) we respectively defined

$$\mathbf{D}\pi(D) = d\pi(D) + V^a \wedge \pi_a(V) - K^a \wedge \pi_a(K) - \tfrac{1}{2}\bar{\varphi} \wedge \pi(\varphi) + \tfrac{1}{2}\bar{\xi} \wedge \pi(\xi) \quad (4.6a)$$

$$\mathbf{D}\pi(A) = d\pi(A) + \tfrac{3}{4} i\bar{\varphi} \wedge \gamma_5 \pi(\varphi) - \tfrac{3}{4} i\bar{\xi} \wedge \gamma_5 \pi(\xi) \quad (4.6b)$$

To arrive at equations (4.5) we made the following prescription on the constraints:

$$\Phi_{A|\Sigma} = \psi_A \approx 0 \quad (4.7)$$

A detailed discussion about the role of the constraints is given in Foussats and Zandron (1991).

Finally, we note that in equations (4.5) the expressions enclosed in the brackets are weakly zero quantities because they are given by the corresponding equation of motion plus a linear combination of the primary second-class constraints ψ_A. Moreover, it is possible to show that the constraints (4.5) are first class (Foussats *et al.*, 1992) because the form-brackets between them gives weakly zero quantities. This shows how, starting from the CCF, the Hamiltonian of the system directly appears written as an appropriate linear combination of first-class constraints. Of course, an alternative way to write the Hamiltonian (4.4) as a suitable linear combination of constraints is to use the decomposition of the conformal gauge fields (K^a, φ) and the Lagrange multipliers (t^{ab}, λ) in the coframe (V^a, ξ), a result obtained in the group manifold formalism.

The very different treatment of the conformal supergravity with respect to the OPS appears when we have to treat the second-order CVF. In the next section we will analyze this question.

5. SECOND-ORDER CVF

We begin by asserting two peculiarities of the CCF:

(i) As already pointed out, the CCF is not a proper Hamiltonian formalism. It does not define a standard mechanical system because it does not really propagate data defined on an initial hypersurface Σ. The generator of time evolutions (or proper Hamiltonian) is $\tilde{\mathscr{H}}$, as seen above.

(ii) The Poisson brackets introduced in the CVF yields more information than the form-brackets (Foussats and Zandron, 1991).

Even so, in the OPS, by starting from the CCF, the second-order Hamiltonian formalism can be recovered by projecting on Σ the total Hamiltonian H_T and the remaining quantities defined in the CCF. This is possible because the OPS is not a higher-derivative theory and further in the CVF the vanishing of the time components of the momenta occurs naturally. Once in the OPS one finds an expression equivalent to that given in (4.4); to arrive at the second order, the torsion equation of motion must be used to write

$\omega_\mu^{ab} = \omega_\mu^{ab}(e, \xi)$. Furthermore, in the OPS, in both the CVF and in the CCF it is possible to define a second-order momentum (Deser *et al.*, 1977; Nelson and Teitelboim, 1977, 1978; Pilati, 1978; Castellani *et al.*, 1982; Henneaux, 1983, 1985; Diaz, 1986; D'Adda, *et al.*, 1985; Lerda *et al.*, 1985, 1987; Foussats and Zandron, 1991), that is canonical conjugate to the graviton field ${}^4e_{a\mu}$. This momentum has vanishing time component and its spatial components are related to the spatial components of the first-order momentum $\Pi_a{}^i(e)$ (D'Adda *et al.*, 1985; Lerda *et al.*, 1985, 1987; Foussats and Zandron, 1991).

In the conformal supergravity case, as it is a higher-derivative theory, to define momenta in the second-order formalism it is necessary to make an Ostrogradski transformation (Nesterenko, 1989; Li, 1990, 1991; Li and Xin, 1991). In this way, nonzero time components of the momenta are obtained. Therefore, for a higher-derivative theory, the second-order CVF cannot be recovered from the CCF.

We turn to the Lagrangian density (3.8) and we consider reduced forms (i.e., forms defined on the coset manifold $M = G/H$). To write the second-order Lagrangian density in components and only containing the physical fields ${}^4e_{a\mu}$ (graviton), ξ_μ (Q-gravitino), and A_μ (axion) we proceed as follows (Kaku *et al.*, 1978; van Nieuwenhuizen, 1981):

(a) The equation of motion (or constraints) $R^a(V) = 0$ can be solved algebraically to obtain the expression of the spin connection:

$$\begin{aligned}\omega_\mu{}^{ab}(e, \xi, D) = &-\omega_\mu{}^{ab}(e) + (D^a\, {}^4e^b{}_\mu - D^b\, {}^4e^a{}_\mu) \\ &+ \tfrac{1}{4}(\bar{\xi}_\mu \gamma^b \xi^a - \bar{\xi}_\mu \gamma^a \xi^b - \bar{\xi}^a \gamma_\mu \xi^b) \end{aligned} \tag{5.1}$$

(b) The equation of motion (or constraints) $\gamma^\mu R_{\mu\nu}(Q) = 0$ can be solved algebraically for the conformal gravitino φ_μ:

$$\varphi_\mu = \tfrac{1}{3}\gamma^\nu (S_{\mu\nu} + \tfrac{1}{4}\gamma_5\, {}^*S_{\mu\nu}) \tag{5.2}$$

where

$$S_{\mu\nu} = (\mathscr{D}_\nu \xi_\mu + \tfrac{1}{2} D_\nu \xi_\mu - \tfrac{3}{4} i A_\nu \gamma_5 \xi_\mu) - (\mu \leftrightarrow \nu) \tag{5.3}$$

${}^*S_{\mu\nu}$ is the dual of S and $\mathscr{D}$ is the Lorentz covariant derivative.

(c) The field equation of motion for the proper conformal gauge field $K_{a\mu}$ is algebraic, so that $K_{a\mu}$ can be eliminated:

$$\begin{aligned}K_{a\mu} = &-\tfrac{1}{4}[\mathscr{R}_{\nu\mu}(M) - \tfrac{1}{6} g_{\mu\nu} \mathscr{R}(M)]\, {}^4e_a{}^\nu + \tfrac{1}{8}\gamma_\mu R_{\alpha\nu}(Q) \xi^\alpha\, {}^4e_a{}^\nu \\ &- \tfrac{1}{16} i\, {}^*R_{\nu\mu}(A)\, {}^4e_a{}^\nu \end{aligned} \tag{5.4}$$

where $\mathscr{R}_{\nu\mu}(M) = R_{\nu\mu}(M)$ for $K_{a\mu} = 0$.

Inserting equation (5.4) in the Lagrangian density (3.8), we find the expression (2.8) of Kaku *et al.* (1978). It is possible to show that the dilaton field D_μ reduces to the Lagrangian density. Therefore, once equations (5.1) and (5.2) are used, the remaining fields are the only physical ones. It is sufficient to analyze the first terms of the Lagrangian (2.8) of Kaku *et al.* (1978) to see that the second-order CVF must be constructed in a very different way than is done in the OPS case.

We do not write all the final expressions for the constraints, but give here only the constructive method and the conclusions.

Looking at the first three terms of the Lagrangian under consideration, i.e.,

$$e[R^{\mu\nu}(M)R_{\nu\mu}(M)-\tfrac{1}{3}R^2(M)]+4\varepsilon^{\mu\nu\rho\sigma}\bar{\varphi}_\rho\gamma_5\gamma_\sigma\mathscr{D}_\nu\varphi_\mu+\cdots \tag{5.5}$$

we can see that these terms contain second-time derivatives on the vierbein components ${}^4e_{a\mu}$ and on the gravitino field components ξ_μ. So we are in the presence of a constrained system with a singular higher-order Lagrangian. It is reasonable to try to work as closely as possible to the Dirac (1964) conjectures for the usual constrained systems. Consequently, we define the following independent dynamical field variables:

$${}^4e_{a\mu}=({}^4e_{ai},\,{}^4e_{a0}=n_aN^\perp+N^ie_{ai}) \tag{5.6a}$$

$$b_{a\mu}=\partial_0\,{}^4e_{a\mu} \tag{5.6b}$$

$$\xi_\mu \tag{5.6c}$$

$$\eta_\mu=\partial_0\xi_\mu \tag{5.6d}$$

$$A_\mu \tag{5.6e}$$

The Ostrogradski transformation (Nesterenko, 1989; Li, 1990, 1991; Li and Xin, 1991) introduces, respectively, the following canonical momenta:

$$\overset{(1)}{\Pi}{}_a{}^\mu=\frac{\partial\mathscr{L}}{\partial b^a{}_\mu}-\partial_\nu\left\{\frac{\partial\mathscr{L}}{\partial(\partial_\nu b^a{}_\mu)}\right\} \tag{5.7a}$$

$$\overset{(2)}{\Pi}{}_a{}^\mu=\frac{\partial\mathscr{L}}{\partial(\partial_0 b^a{}_\mu)} \tag{5.7b}$$

$$\overset{(1)}{\Pi}{}^\mu=\frac{\partial\mathscr{L}}{\partial\bar{\eta}_\mu}-\partial_\nu\left\{\frac{\partial\mathscr{L}}{\partial(\partial_\nu\bar{\eta}_\mu)}\right\} \tag{5.7c}$$

$$\overset{(2)}{\Pi}{}^\mu=\frac{\partial\mathscr{L}}{\partial(\partial_0\bar{\xi}_\mu)} \tag{5.7d}$$

$$\Pi^\mu(A)=\frac{\partial\mathscr{L}}{\partial(\partial_0 A_\mu)} \tag{5.7e}$$

where the Poisson brackets for canonical conjugate variables are given by

$$\left[{}^4e^a{}_\mu(x), \overset{(1)}{\Pi}{}_b{}^\nu(y)\right] = -\left[\overset{(1)}{\Pi}{}_b{}^\nu(y), {}^4e^a{}_\mu(x)\right] = \delta^a{}_b\, \delta^\mu{}_\nu\, \delta(x-y) \tag{5.8a}$$

$$\left[b^a{}_\mu(x), \overset{(2)}{\Pi}{}_b{}^\nu(y)\right] = -\left[\overset{(2)}{\Pi}{}_b{}^\nu(y), b^a{}_\mu(x)\right] = \delta^a{}_b\, \delta^\mu{}_\nu\, \delta(x-y) \tag{5.8b}$$

$$\left[\bar{\xi}_\mu{}^{(\alpha)}(x), \overset{(1)}{\Pi}{}^\nu_{(\beta)}(y)\right] = \left[\overset{(1)}{\Pi}{}^\nu_{(\beta)}(y), \bar{\xi}_\mu{}^{(\alpha)}(x)\right] = \delta^{(\alpha)}_{(\beta)}\, \delta^\nu{}_\mu\, \delta(x-y) \tag{5.8c}$$

$$\left[\bar{\eta}_\mu{}^{(\alpha)}(x), \overset{(2)}{\Pi}{}^\nu_{(\beta)}(y)\right] = \left[\overset{(2)}{\Pi}{}^\nu_{(\beta)}(y), \bar{\eta}_\mu{}^{(\alpha)}(x)\right] = \delta^{(\alpha)}_{(\beta)}\, \delta^\nu{}_\mu\, \delta(x-y) \tag{5.8d}$$

$$[A_\mu(x), \Pi^\nu(y)] = -[\Pi^\nu(y), A_\mu(x)] = \delta^\nu{}_\mu\, \delta(x-y) \tag{5.8e}$$

Using these definitions and relations, one can pass from the Lagrangian formalism to the second-order CVF. We give here the conclusions, but we do not write the explicit results.

1. In this higher-derivative singular system the canonical Hamiltonian is written

$$\mathscr{H}_{\text{can}} = b^a{}_\mu \overset{(1)}{\Pi}{}_a{}^\mu + \dot{b}^a{}_\mu \overset{(2)}{\Pi}{}_a{}^\mu + \bar{\eta}_\mu \overset{(1)}{\Pi}{}^\mu + \dot{\bar{\eta}}_\mu \overset{(2)}{\Pi}{}^\mu + \dot{A}_\mu \Pi^\mu(A) - \mathscr{L} \tag{5.9}$$

where $\mathscr{L}$ is only a functional of the field variables (5.6) and their first space-time derivative, and we replaced ${}^4\dot{e}_{a\mu}$ and $\dot{\bar{\xi}}_\mu$ by $b_{a\mu}$ and $\bar{\eta}_\mu$, respectively. Once the Lagrangian density is explicitly used the velocities $\dot{b}^a{}_\mu$ and $\dot{\xi}_\mu$ are eliminated from $\mathscr{H}_{\text{can}}$. Of course, the fields $b_{a\mu}$ and η_μ cannot be eliminated from the formalism.

2. The system has a set $\{\Omega^{(k)}_a\}$ of bosonic and fermionic primary constraints and therefore the total (or extended) Hamiltonian density is given by

$$\tilde{\mathscr{H}}_T = \mathscr{H}_{\text{can}} + \lambda^c_{(k)} \Omega^{(k)}_c \tag{5.10}$$

where the Lagrange multipliers $\lambda^c_{(k)}$ can be evaluated by means of the Hamilton equation $\dot{A} = [A, \mathscr{H}_T]_{\text{PB}}$ with $\mathscr{H}_T = \int d^3x\, \tilde{\mathscr{H}}_T$.

3. From the stationarity of the primary constraints, the secondary constraints are found according to the Dirac algorithm:

$$\Omega^{(k)}_a = [\Omega^{(k-1)}_a, \mathscr{H}_T]_{\text{PB}} \tag{5.11}$$

This algorithm must be continued until $\Omega_b^{(k)}$ satisfies

$$\Omega_b^{(k+1)} = [\Omega_b^{(k)}, \mathscr{H}_T]_{\rm PB} = C^a_{bn}\Omega_a^{(n)} \tag{5.12}$$

4. It is also possible to conclude that the system in the second-order formalism has primary and secondary constraints. This set contains constraints of both first and second class. Each first-class constraint corresponds to a gauge invariance of the theory under a local gauge transformation.

The presence of second-class constraints makes it necessary to follow the Dirac (1964) conjectures, i.e., the Dirac brackets must be defined from the Poisson brackets. Then, as is usual in constrained Hamiltonian systems, the second-class constraints must be taken strongly equal to zero and so they can be eliminated from the formalism.

6. CONCLUSIONS

The $N=1$, $d=4$ conformal supergravity can be formulated in the picture of the first-order CCF, starting from the Lagrangian formalism on the group manifold. The primary second-class constraint Φ_A allows us to write the total Hamiltonian H_T because there are no secondary constraints in the formalism. By making the space-time decomposition and by straightforward computation it is possible to find the proper Hamiltonian $\tilde{\mathscr{H}}$, as a generator of time evolutions, as a suitable linear combination of the set of first-class constraints $\mathscr{H}_A(x)$. As the superconformal system is described by a higher-derivative Lagrangian, the second-order CVF is nontrivial and its construction is very different from that given for the OPS. An Ostrogradski transformation is needed to introduce canonical momenta. In dimension $d=4$ it is necessary to perform the Ostrogradski transformation on both the bosonic $({}^4e_{a\mu})$ and the fermionic (ξ_μ) fields. We notice that in the $d=3$ conformal theory (Foussats and Zandron, 1991) it is necessary to perform such a transformation only on the bosonic field because the corresponding supersymmetric Chern–Simons Lagrangian does not contain second-time derivatives on the fermionic field ξ_μ. Finally, it can be shown that in the second-order Hamiltonian formalism there are primary and secondary constraints and they are of first and second class. The appearance of second-class constraints, characteristic of (super)gravity, makes it necessary to follow Dirac's (1964) conjectures and so the Dirac brackets must be introduced.

ACKNOWLEDGMENTS

The authors are Fellows of the Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina, which they thank for financial support.

REFERENCES

Arnowitt, R., Deser, S., and Misner, C. W. (1982). In *Gravitation: An Introduction to Current Research*, L. Witten, ed., Wiley, New York.

Castellani, L., Fré, P., and van Nieuwenhuizen, P. (1981). *Annals of Physics*, **136**, 398.

Castellani, L., van Neuwenhuizen, P., and Pilati, M. (1982). *Physical Review D*, **26**, 352.

Castellani, L., D'Auria, R., and Fré, P. (1991). *Supergravity and Superstrings: A Geometric Perspective*, World Scientific, Singapore.

D'Adda, A., Nelson, J. E., and Regge, T. (1985). *Annals of Physics*, **165**, 384.

Diaz, A. (1986). *Physical Review D*, **33**, 2801, 2809.

Dirac, P. A. M. (1964). *Lectures on Quantum Mechanics*, Yeshiva University Press, New York.

Deser, S., Kay, J. H., and Stelle, K. S. (1977). *Physical Review D*, **16**, 2448.

Foussats, A., and Zandron, O. (1990). *International Journal of Modern Physics A*, **5**, 725.

Foussats, A., and Zandron, O. (1991). *Physical Review D*, **43**, 1883.

Foussats, A., Repetto, C., Zandron, O. P., and Zandron, O. S. (1992). *Classical and Quantum Gravity*, **9**, 2217.

Henneaux, M. (1983). *Physical Review D*, **27**, 986.

Henneaux, M. (1986). *Physics Letters*, **168B**, 233.

Kaku, M., Townsend, P. K., and van Nieuwenhuizen, P. (1978). *Physics Review D*, **17**, 3179.

Lerda, A., Nelson, J. E., and Regge, T. (1985). *Physics Letters*, **161B**, 294.

Lerda, A., Nelson, J. E., and Regge, T. (1987). *International Journal of Modern Physics*, **A2**, 1843.

Li, Zi-ping (1990). *International Journal of Theoretical Physics*, **29**, 165.

Li, Zi-ping (1991). *Journal of Physics A: Mathematical and General*, **24**, 4261.

Li, Zi-Ping, and Xin, Li (1991). *International Journal of Theoretical Physics*, **30**, 225.

Nelson, J. E., and Teitelboim, C. (1977). *Physics Letters*, **69B**, 81.

Nelson, J. E., and Teitelboim, C. (1978). *Annals of Physics*, **116**, 1.

Nesterenko, V. V. (1989). *Journal of Physics A: Mathematical and General*, **22**, 1673.

Pilati, M. (1978). *Nuclear Physics B*, **132**, 138.

Salam, A., and Sezgin, E., eds. (1989). *Supergravities in Diverse Dimensions*, North-Holland/World Scientific, Amsterdam, Vol. 2.

Van Nieuwenhuizen, P. (1981). *Physics Reports*, **68**, 189–398.

Van Nieuwenhuizen, P. (1985). *Physical Review D*, **32**, 872.